Integral domains with only finitely many star operations

Abstract. Recall that a nonzero ideal I of a domain R is said to be *divisorial* if $(I^{-1})^{-1} = I$ (where $I^{-1} = (R : I) = \{x \in \mathrm{qf}(R) \mid xI \subseteq R\}$). Heinzer showed that an integrally closed domain R has all nonzero ideals divisorial if and only if R is an h-local Prüfer domain with invertible maximal ideals; and Bass and Matlis showed that a Noetherian domain has all ideals divisorial if and only if R has Krull dimension one and M^{-1} is a 2-generated R-module for each maximal ideal M of R. Now recall that a star operation on a domain R is a map $*$ from the set of nonzero fractional ideals of R to itself such that, for all $a \in \mathrm{qf}(R)$ and nonzero fractional ideals I, J of R, we have (1) $(aI)^* = aI^*$ and $(a)^* = (a)$, (2) $I \subseteq I^*$ and $I \subseteq J$ implies $I^* \subseteq J^*$, and (3) $(I^*)^* = I^*$. Simple examples include the d-operation ($I^d = I$ for all I) and the v-operation ($I^v = (I^{-1})^{-1}$). One sees easily that $d \leq * \leq v$ (that is, $I \subseteq I^* \subseteq I^v$ for all I). Hence R has all ideals divisorial if and only if R admits only one star operation. Thus it is natural to attempt to characterize domains that admit only finitely many star operations in the integrally closed and Noetherian cases. In this talk we discuss what is currently known about this problem.